

EECS 312 Discussion 8

10/29/10

Myung-Chul Kim
(mckima@umich.edu)

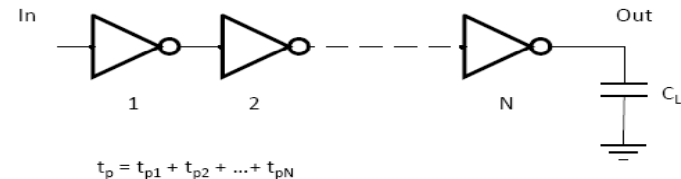
Overview

- Reminder
 - Second section of Lab 3: Due 2 Nov (Tue.)
- Lab 3 Part 2 Help
- Optimal Buffering
- Exercise

Lab 3 Part 2

- Input to the system: a' and b' (ideal step transitions)
 - Your design take a and b as inputs
- $E_{\text{SUPPLY}} = \int P(t)dt = V_{\text{DD}} \int_{t_1}^{t_2} I_{\text{SUPPLY}}(t)dt$
 - t1, t2: from 10% input transition to 90% output transition (crossing over 0.25V or 2.25V)
 - I_{SUPPLY} can be measured at your VDC
 - Use “Intel” function in the Waveform Calculator
 - Identify the highest energy consumption input transition

Optimal Buffering

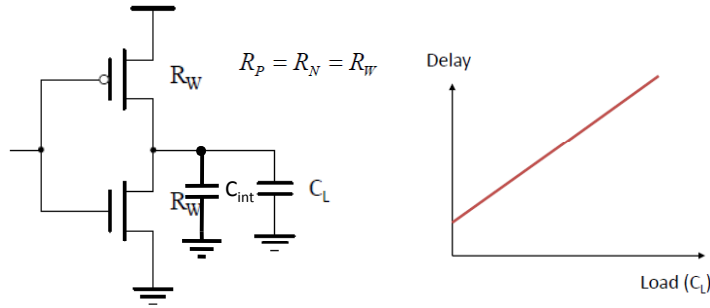


- Objective : Sizing for Performance
- Constraint: Self-loading and Intrinsic Delay
- If C_L is given:
 - How many stages are needed to minimize the delay?
 - How to size the inverters for a given number of stages?

Inverter Intrinsic Delay

- Assume that for $W_p = 2W_n = 2W_{unit}$
 - same pull-up and pull-down currents
 - approx. equal resistances $R_N = R_P$
 - approx. equal rise t_{pLH} and fall t_{pHL} delays

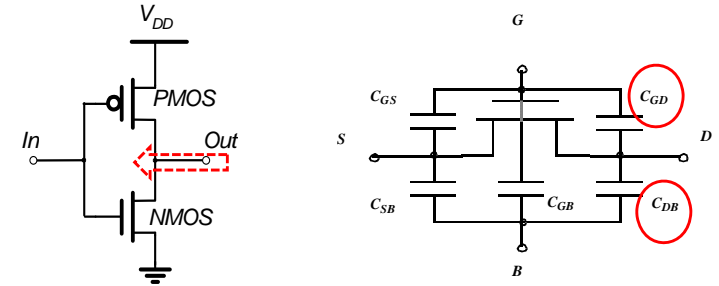
$$t_{pHL} = (\ln 2) R_N C_L \quad t_{pLH} = (\ln 2) R_P C_L$$



$$\text{Delay} = 0.69R_W(C_{int} + C_L) = 0.69R_W C_{int} + 0.69R_W C_L$$

= Delay (Internal) + Delay (Load)

Inverter Intrinsic Capacitances



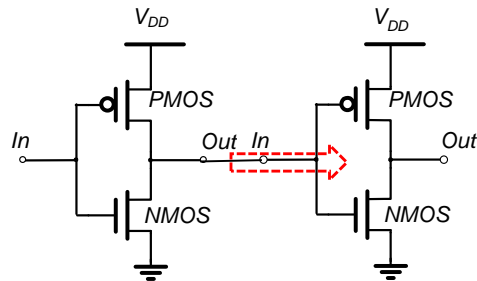
$$C_{GD} = C_{GCD} + C_{GD-Overlap} = C_{GD-Overlap}$$

but with Miller Effect (will be covered) $\rightarrow 2C_{GD}$

$$C_{DB} = K_{eq} * (C_{JD}W + C_{JW}(2L_D + W))$$

Details will be covered in class soon

Inverter Gate Load Capacitances

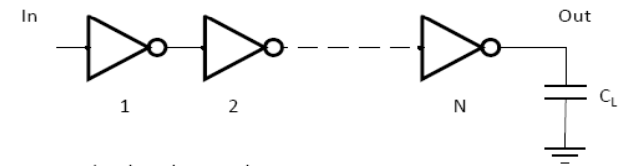


$$C_G = C_{GC} + C_{GD-Overlap} + C_{GS-Overlap}$$

$$= C_{ox} L_{eff} W + C_{GDO} W + C_{GSO} W$$

Optimal Tapering for Given N

- Stage Delay $= (\ln 2) \cdot R_W (C_{int} + C_L) = (\ln 2) \cdot R_W C_{int} (1 + C_L / C_{int}) = t_{p0} (1 + f / \gamma)$
 - $f = C_L / C_g$ - effective fan out, $C_{int} = \gamma C_g$



$$t_p = t_{p1} + t_{p2} + \dots + t_{pN}$$

$$t_{pj} = t_{p0} \left(1 + \frac{C_{g,j+1}}{\gamma C_{g,j}} \right)$$

$$t_p = \sum_{j=1}^N t_{p,j} = t_{p0} \sum_{i=1}^N \left(1 + \frac{C_{g,j+1}}{\gamma C_{g,j}} \right), \quad C_{g,N+1} = C_L$$

Optimal Tapering for Given N

- Tapering factor between stages $f = F^{1/N}$
 - F = overall effective fan out = C_L / C_{g1}
 - Each stage has the same effective fanout
 - $C_{g,j+1}/C_{g,j} = C_{g,j}/C_{g,j-1} \rightarrow f_j = f_{j-1}$
 - Size of each stage is the geometric mean of two neighbors
 - $C_{g,j} = \sqrt{C_{g,j-1} C_{g,j+1}}$
- Minimum path delay
 - $t_p = N * t_{p0}(1+f/\gamma)$
 - Each stage has the same delay

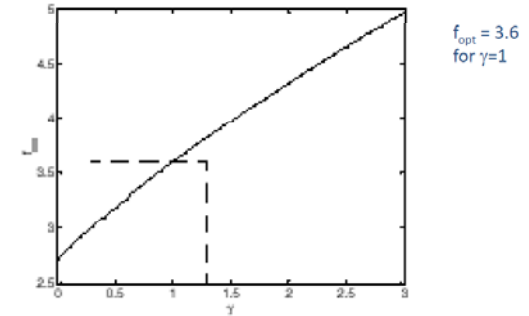
Optimum Number of Stages for a Given F

- Minimum Path Delay: $t_p = N * t_{p0}(1 + F^{1/N} / \gamma)$

- Optimum Number of Stages:

N is solution of $\gamma + \frac{N\sqrt[N]{F}}{N} - \frac{N\sqrt[N]{F} \ln F}{N} = 0$

or $N = \ln F / \ln f$ where f is given by $f = \exp(1 + \gamma/f)$



Buffer design example

Diagram	N	f	t_p
	1	64	$65t_{p0}$
	2	8	$18t_{p0}$
	3	4	$15t_{p0}$
	4	2.8	$15.3t_{p0}$

Exercise

- A 2λ wide inverter with input capacitance C_m must drive a load of $50C_m$. $\gamma = 0.5$. The output signal may be inverted. Derive the optimum delay, # of stages, and sizing.