EECS 312: Digital Integrated Circuits
Midterm Exam

12 March 2009

Robert Dick

Show your work. Derivations are required for credit; end results are insufficient.
Closed book. No electronic mental aids.

Honor Pledge: I have neither given nor received aid in this exam.

Signature:

1. **(5 pts.)** $k_n = \frac{W \mu_n \epsilon_{ox}}{L t_{ox}}$ When the move from SiO_2 to high-κ gate dielectric occurred, what qualitative changes were made to the variables upon which k_n depends? Use only a few sentence fragments for your answer.

2. **(10 pts.)** Consider a static CMOS gate implementing the following function:

$$f(a, b, c) = ab + \bar{b}c$$

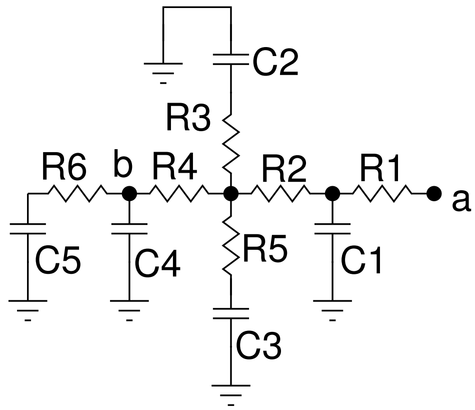
   This gate is subjected to the following transition $a = 1, b = 0, c = 1 \rightarrow a = 1, b = 1, c = 1$. Will the gate's dynamic energy consumption be greater if its inputs come from other static static CMOS gates and inverters, or from DCVSL gates? Use one sentence to explain why. You may also use a schematic if that makes your answer clearer.

3. **(10 pts.)** Given process variation resulting in a Gaussian distribution of threshold voltage around its nominal value, will total integrated circuit sub-threshold leakage power consumption be higher, the same, or lower than that of an integrated circuit in which all transistors have nominal threshold voltages? Use at most two sentences to explain why.

4. **(10 pts.)** Determine the high-to-low propagation delay for an inverter with a 500 nm wide NMOSFET and a 1 µm wide PMOSFET with its output connected to another identical inverter. Assume the default 250 nm process. Do consider overlap capacitance. Recall that it is necessary to consider both bottom and sidewall capacitance. You may neglect the resistance and capacitance of the wire connecting the two inverters. Note that the switch model reference table gives values for $W/L =$ transistors. Show your work.

5. **(10 pts.)** Assuming the default 250 nm process, determine the minimum and maximum percentage change in subthreshold leakage current of a minimal-width NMOSFET given that the threshold voltage may deviate by 0.1 V from the nominal in either direction. You may assume that $n = 1.5$.

6. **(10 pts.)** Determine the Elmore delay from Node a to Node b in the following circuit.

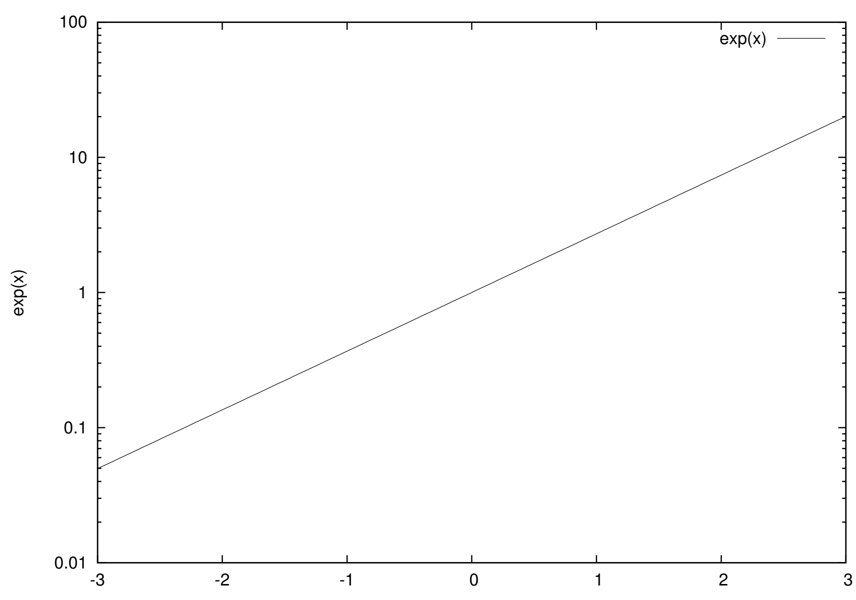


7. **(10 pts.)** Show the circuit diagram for a static CMOS implementation of the following function sized to have the same resistance to ground and V_{DD} as an inverter with a W wide NMOSFET and an $2W$ wide PMOSFET. You may assume access to complemented and uncomplemented input literals, i.e., a and $\overline{a}$.

$$f(a, b, c) = (ab + \overline{a}\,\overline{b})c$$

8. **(0 pts.)** Was this exam ☐ tricky and surprising or did it ☐ fairly closely follow the material we focused on in class?

	C_{OX} (fF/µm²)	C_O (fF/µm)	C_j (fF/µm²)	m_j	ϕ_b (V)	C_{jsw} (fF/µm)	m_{jsw}	ϕ_{bsw} (V)
NMOS	6	0.31	2	0.5	0.9	0.28	0.44	0.9
PMOS	6	0.27	1.9	0.48	0.9	0.22	0.32	0.9



MODELS FOR CMOS DEVICES

CMOS (0.25 μm) – Unified Model.

	V_{T0} (V)	γ ($V^{0.5}$)	V_{DSAT} (V)	k' (A/V^2)	λ (V^{-1})
NMOS	0.43	0.4	0.63	115×10^{-6}	0.06
PMOS	–0.4	–0.4	–1	-30×10^{-6}	–0.1

CMOS (0.25 μm) – Switch Model (R_{eq})

V_{DD} (V)	1	1.5	2	2.5
NMOS (kΩ)	35	19	15	13
PMOS (kΩ)	115	55	38	31

CMOS (0.25 μm) – BSIM Model

See Website: *http://bwrc.eecs.berkeley.edu/IcBook*

VALUES OF MATERIAL AND PHYSICAL CONSTANTS

Name	Symbol	Value	Units
Room temperature	T	300 (= 27°C)	K
Boltzman constant	k	1.38×10^{-23}	J/K
Electron charge	q	1.6×10^{-19}	C
Thermal voltage	$\phi_T = kT/q$	26	mV (at 300 K)
Intrinsic Carrier Concentration (Silicon)	n_i	1.5×10^{10}	cm^{-3} (at 300 K)
Permittivity of Si	ε_{si}	1.05×10^{-12}	F/cm
Permittivity of SiO_2	ε_{ox}	3.5×10^{-13}	F/cm
Resistivity of Al	ρ_{Al}	2.7×10^{-8}	Ω-m
Resistivity of Cu	ρ_{Cu}	1.7×10^{-8}	Ω-m
Magnetic permeability of vacuum (similar for SiO_2)	μ_0	12.6×10^{-7}	Wb/Am
Speed of light (in vacuum)	c_0	30	cm/nsec
Speed of light (in SiO_2)	c_{ox}	15	cm/nsec

FORMULAS AND EQUATIONS

Diode

$$I_D = I_S(e^{V_D/\phi_T} - 1) = Q_D/\tau_T$$

$$C_j = \frac{C_{j0}}{(1 - V_D/\phi_0)^m}$$

$$K_{eq} = \frac{-\phi_0^m}{(V_{high} - V_{low})(1-m)} \times [(\phi_0 - V_{high})^{1-m} - (\phi_0 - V_{low})^{1-m}]$$

MOS Transistor

$$V_T = V_{T0} + \gamma(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|})$$

$$I_D = \frac{k'_n}{2}\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) \text{ (sat)}$$

$$I_D = \upsilon_{sat} C_{ox} W\left(V_{GS} - V_T - \frac{V_{DSAT}}{2}\right)(1 + \lambda V_{DS}) \text{ (velocity sat)}$$

$$I_D = k'_n \frac{W}{L}\left((V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right) \text{ (triode)}$$

$$I_D = I_S e^{\frac{V_{GS}}{nkT/q}}\left(1 - e^{-\frac{V_{DS}}{kT/q}}\right) \text{ (subthreshold)}$$

Deep Submicron MOS Unified Model

$I_D = 0$ for $V_{GT} \le 0$

$$I_D = k'\frac{W}{L}\left(V_{GT}V_{min} - \frac{V_{min}^2}{2}\right)(1 + \lambda V_{DS}) \text{ for } V_{GT} \ge 0$$

with $V_{min} = \min(V_{GT}, V_{DS}, V_{DSAT})$

and $V_{GT} = V_{GS} - V_T$

MOS Switch Model

$$R_{eq} = \frac{1}{2}\left(\frac{V_{DD}}{I_{DSAT}(1 + \lambda V_{DD})} + \frac{V_{DD}/2}{I_{DSAT}(1 + \lambda V_{DD}/2)}\right)$$

$$\approx \frac{3}{4}\frac{V_{DD}}{I_{DSAT}}\left(1 - \frac{5}{6}\lambda V_{DD}\right)$$

Inverter

$$V_{OH} = f(V_{OL})$$

$$V_{OL} = f(V_{OH})$$

$$V_M = f(V_M)$$

$$t_p = 0.69 R_{eq} C_L = \frac{C_L(V_{swing}/2)}{I_{avg}}$$

$$P_{dyn} = C_L V_{DD} V_{swing} f$$

$$P_{stat} = V_{DD} I_{DD}$$

Static CMOS Inverter

$$V_{OH} = V_{DD}$$

$$V_{OL} = GND$$

$$V_M \approx \frac{rV_{DD}}{1+r} \quad \text{with} \quad r = \frac{k_p V_{DSATp}}{k_n V_{DSATn}}$$

$$V_{IH} = V_M - \frac{V_M}{g} \qquad V_{IL} = V_M + \frac{V_{DD} - V_M}{g}$$

$$\text{with } g \approx \frac{1+r}{(V_M - V_{Tn} - V_{DSATn}/2)(\lambda_n - \lambda_p)}$$

$$t_p = \frac{t_{pHL} + t_{pLH}}{2} = 0.69 C_L\left(\frac{R_{eqn} + R_{eqp}}{2}\right)$$

$$P_{av} = C_L V_{DD}^2 f$$

Interconnect

Lumped *RC*: $t_p = 0.69\, RC$

Distributed *RC*: $t_p = 0.38\, RC$

RC-chain:

$$\tau_N = \sum_{i=1}^{N} R_i \sum_{j=i}^{N} C_j = \sum_{i=1}^{N} C_i \sum_{j=1}^{i} R_j$$

Transmission line reflection:

$$\rho = \frac{V_{refl}}{V_{inc}} = \frac{I_{refl}}{I_{inc}} = \frac{R - Z_0}{R + Z_o}$$

CMOS COMBINATIONAL LOGIC

Transistor Sizing using Logical Effort

$$F = \frac{C_L}{C_{g1}} = \prod_1^N \frac{f_i}{b_i} \qquad G = \prod_1^N g_i \qquad D = t_{p0}\sum_{j=1}^{N}\left(p_j + \frac{f_j g_j}{\gamma}\right)$$

$$B = \prod_1^N b_i \qquad H = FGB \qquad D_{min} = t_{p0}\left(\sum_{j=1}^{N} p_j + \frac{N(\sqrt[N]{H})}{\gamma}\right)$$