Homework 3 73

1)a)

Cube	Kernel
B	AC + CD + AD
C	AB + BD + DE
D	BC + AB + CE
AB	C + D
BC	A + D
BD	C + A
CD	B + E

b) initial # of literals: 12

$L = AB(C+D) + BCD + CDE$

$L = AB(C+D) + CD(B+E)$

final # of literals: 8

2)a)

$F = A\bar{B}\bar{C} + \bar{A}\bar{C} + \bar{A}B$

$= A\bar{B}\bar{C} + \bar{A}\bar{C}(1+\bar{B}) + \bar{A}B$

$= A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{C} + \bar{A}B$

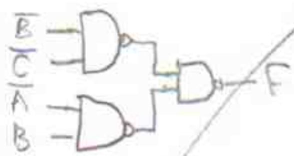
$= (A+\bar{A})\bar{B}\bar{C} + \bar{A}\bar{C}(B+\bar{B}) + \bar{A}B$

$= \bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}B$

$= \bar{B}\bar{C}(1+\bar{A}) + \bar{A}B(1+\bar{C})$

$= \overline{\overline{\bar{B}\bar{C} + \bar{A}B}}$

$= \overline{\overline{(\bar{B}\bar{C})}\,\overline{(\bar{A}B)}}$



6 PMOS
6 NMOS

b)

$F = \bar{B}\bar{C} + \bar{A}B$

$= \bar{B}\bar{C}(1+\bar{A}) + \bar{A}B(1+\bar{C})$

$= \bar{B}\bar{C} + \bar{A}B + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}$

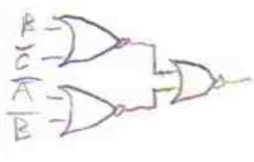
$= \bar{B}\bar{C} + \bar{A}B + \bar{A}\bar{C}(\bar{B}+B)$

$= \bar{B}\bar{C} + \bar{A}B + \bar{A}\bar{C} + B\bar{B}$

$= \bar{C}(\bar{A}+\bar{B}) + B(\bar{A}+\bar{B})$

$= \overline{\overline{(B+\bar{C})(\bar{A}+\bar{B})}}$

$= \overline{\overline{(B+\bar{C})} + \overline{(\bar{A}+\bar{B})}}$



6 PMOS
6 NMOS

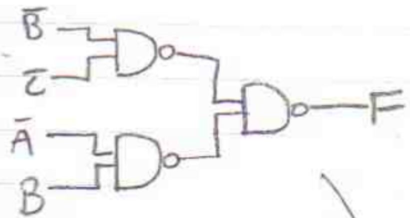
2. $F(A,B,C) = A\bar{B}\bar{C} + \bar{A}\bar{C} + \bar{A}B$

F (A \ BC)	00	01	11	10
0	1		1	1
1	1			

(a)

$F = \bar{B}\bar{C} + \bar{A}B$

$$F = \overline{\overline{\bar{B}\bar{C} + \bar{A}B}} = \overline{(\overline{\bar{B}\bar{C}})(\overline{\bar{A}B})}$$



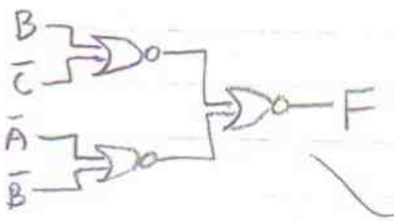
6 PMOS (2 per NAND)
6 NMOS (2 per NAND)

(b)

F (A \ BC)	00	01	11	10
0		1		
1		1	1	1

$\bar{F} = \bar{B}C + AB$

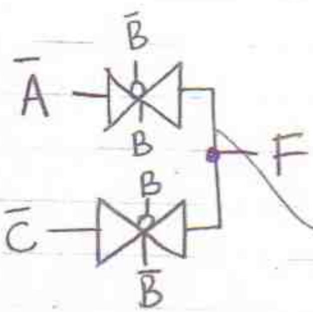
$$F = \overline{\bar{B}C + AB} = \overline{(\overline{B+\bar{C}}) + (\overline{\bar{A}+\bar{B}})}$$



6 PMOS (2 per NOR)
6 NMOS (2 per NOR)

(c)

A	B	C	F	
0	0	0	1	$> \bar{C}$
0	0	1	0	
0	1	0	1	$> \bar{A}$
0	1	1	1	
1	0	0	1	$> \bar{C}$
1	0	1	0	
1	1	0	0	$> \bar{A}$
1	1	1	0	

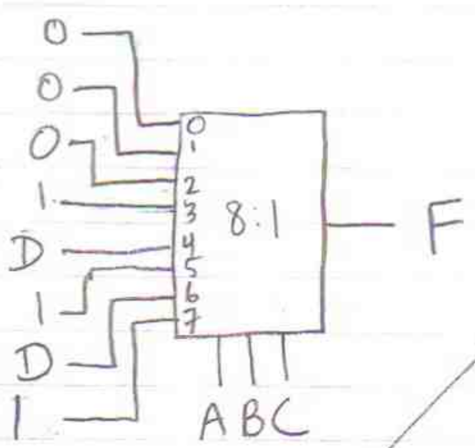


2 PMOS (1 per TG)
2 NMOS (1 per TG)

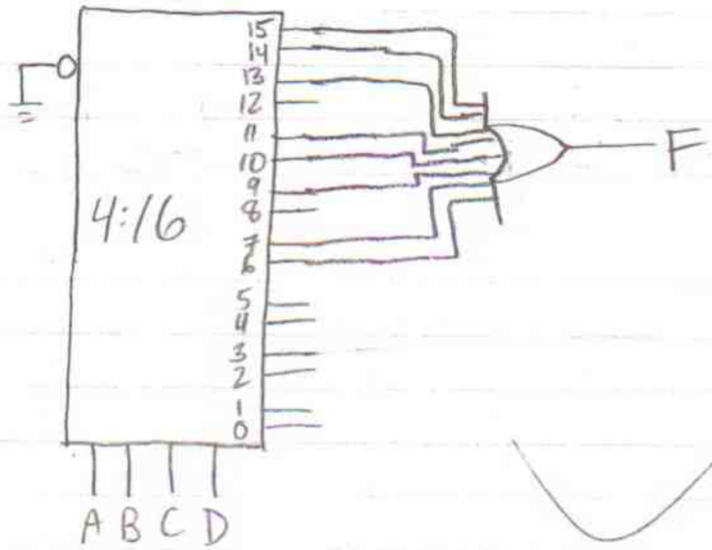
3. $F(A,B,C,D) = \bar{A}BC + AD + AC$

(a)

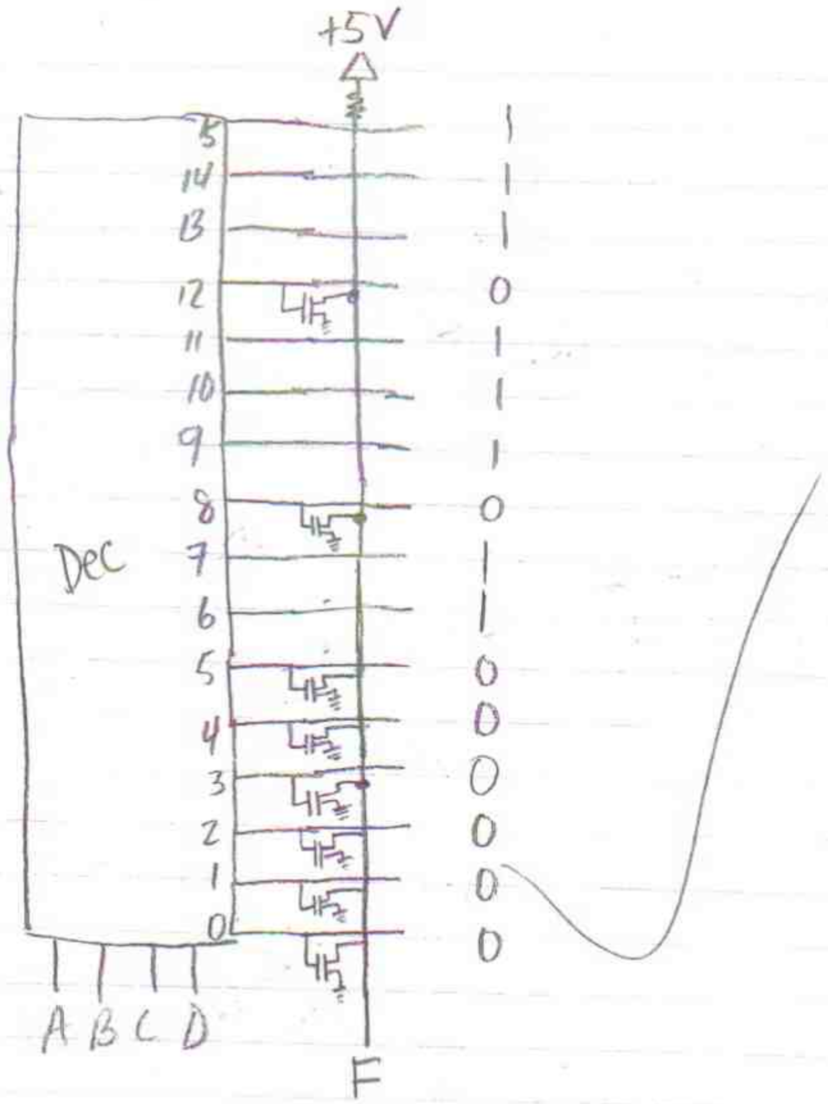
	A	B	C	D	F	
0	0	0	0	0	0	> 0
1	0	0	0	1	0	
2	0	0	1	0	0	> 0
3	0	0	1	1	0	
4	0	1	0	0	0	> 0
5	0	1	0	1	0	
6	0	1	1	0	1	> 1
7	0	1	1	1	1	
8	1	0	0	0	0	> D
9	1	0	0	1	1	
10	1	0	1	0	1	> 1
11	1	0	1	1	1	
12	1	1	0	0	0	> D
13	1	1	0	1	1	
14	1	1	1	0	1	> 1
15	1	1	1	1	1	



(b)



(c)



4. (a) $G = AC + ADE + BC + BDE$ $\boxed{\text{10 literals}}$

kernels: A, B, C, AD, AE, BD, BE, DE

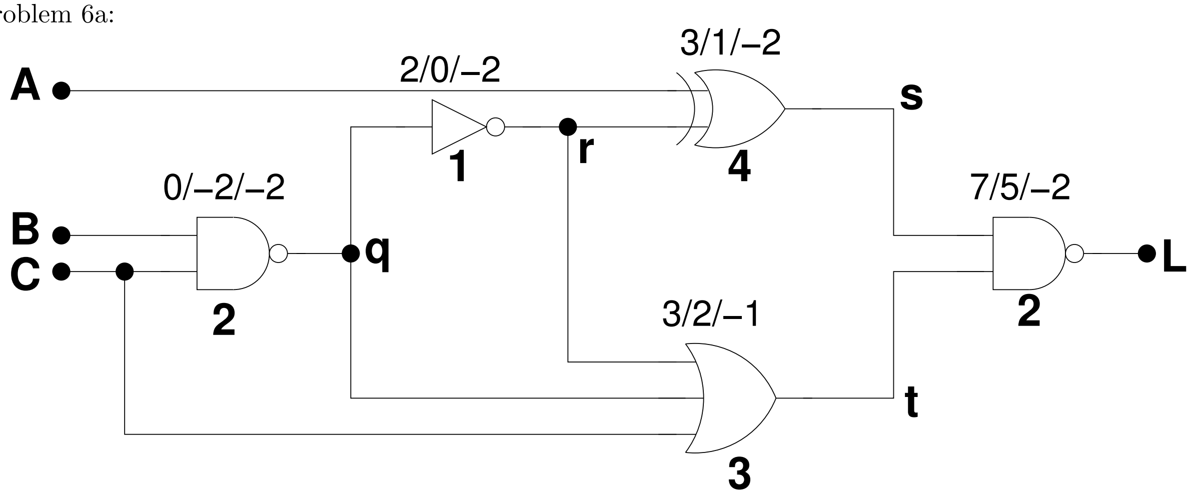
$G = AC + BC + (A+B)DE$

$G = (A+B)C + (A+B)DE$

kernels: $(A+B)$

$G = (A+B)(C+DE)$ $\boxed{\text{5 literals}}$

- Problem 4: See slides for further examples of kernel extraction.
- Problem 5: A *graph* is composed of a set of vertices and edges. Each edge is connected to exactly two vertices. The edges in a *directed graph* are asymmetric: each has a source and destination (indicated by an arrow). A *directed acyclic graph* contains no cycles, i.e., it is impossible to start from any vertex and, following edges in the forward direction only, revisit the same vertex again. A *tree* is a directed acyclic graph containing no reconverging paths.
- Problem 6a:



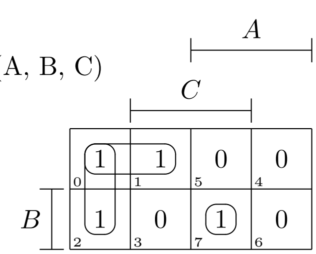
Key: EST/LST/slack

The circuit does not meet its timing constraints.

- Problem 6b:

A B C	L
0 0 0	1
0 0 1	1
0 1 0	1
0 1 1	0
1 0 0	0
1 0 1	0
1 1 0	0
1 1 1	1

L(A, B, C)



$$
\begin{aligned}
L(A,B,C) &= \overline{A}\,\overline{B} + \overline{A}\,\overline{C} + ABC && (1)\\
&= \overline{A}\,(\overline{B} + \overline{C}\,) + A(BC) && (2)\\
&= \overline{A}\,(\overline{BC}) + A(BC) && (3)\\
&= A \oplus \overline{BC} && (4)\\
&= \mathrm{XOR}(A, \mathrm{NAND}(BC)) && (5)
\end{aligned}
$$

Total delay = 6 (NAND2 followed by XOR2).

- Problem 6c: