Number Systems

Homework

Introduction to number systems

Consider a base-10 number: \(1,293\)

\[1,293 = 1 \cdot 10^3 + 2 \cdot 10^2 + 9 \cdot 10^1 + 3 \cdot 10^0\]

For base-10, given an \(n\)-digit number in which \(d_i\) is the \(i\)th digit, the number is

\[\sum_{i=0}^{n} 10^{i-1} \cdot d_i\]

Convert 2,0123 from base-3 to base-10.

\[
\begin{align*}
2 \cdot 3^3 + 0 \cdot 3^2 + 1 \cdot 3^1 + 2 \cdot 3^0 \\
2 \cdot 27 + 0 \cdot 9 + 1 \cdot 3 + 2 \cdot 1 \\
54 + 0 + 3 + 2 \\
59_{10}
\end{align*}
\]

Convert 5910 from base-10 to base-3. Repeatedly divide by the greatest power of \(b\) (the base) that is less than the number.

<table>
<thead>
<tr>
<th>Remainder</th>
<th>Try dividing</th>
<th>Digit</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>37 (\equiv) 81</td>
<td>3</td>
<td>O.K.</td>
</tr>
<tr>
<td>59 (-) 2 \cdot 27 = 5</td>
<td>3 (\equiv) 9</td>
<td>0</td>
<td>Too big</td>
</tr>
<tr>
<td>5 (-) 0 \cdot 9 = 5</td>
<td>3 (\equiv) 3</td>
<td>1</td>
<td>O.K.</td>
</tr>
<tr>
<td>5 (-) 1 \cdot 3 = 2</td>
<td>3 (\equiv) 2</td>
<td>2</td>
<td>O.K.</td>
</tr>
</tbody>
</table>

\(020_3 = 2012_3\)

Useful bases

- 2: Also called binary. Most fundamental base in digital logic. Know this like the back of your hand.
- 8: Also called octal. Sometimes used by programmers. Prefer base 16.
- 10: Also called decimal or Arabic.
- 16: Also called hexadecimal or simple hex. One of the most compact and beautiful representations for digital computer programmers.

Decimal

- Most commonly used by human beings.
- Also called Arabic.
  - Actually developed in India and brought to Europe via Arabian empire.
- Largely replaced Roman numerals, which were more cumbersome when writing the large and complicated numbers used in astronomy and wide-spread trade.
Number systems

- Representation of positive numbers same in most systems
- A few special-purpose alternatives exist, e.g., Gray code
- Alternatives exist for signed numbers

Base-16: Hex

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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</tbody>
</table>

Often prefixed with 0x.
What is 0xFF?

Computer geek culture reference

- Spelling things in ASCII (hex or binary)
- This is one of the lower forms of geek culture, akin to bad puns
- However, at least one university has things written into its buildings with subtle brick patterns in ASCII binary

```
4a6934207375616e34206a6931207368653420
6a69342068656e332068616f332077616e3221
```