Outline

1. Number Systems
2. Homework
Consider a base-10 number: 1,293

$$1,293 = 1 \cdot 10^3 + 2 \cdot 10^2 + 9 \cdot 10^1 + 3 \cdot 10^0$$

For base-10, given an $n$-digit number in which $d_i$ is the $i$th digit, the number is

$$\sum_{i=0}^{n} 10^{i-1} \cdot d_i$$
This works for any base. Convert $2,012_3$ from base-3 to base-10.

\[
2 \cdot 3^3 + 0 \cdot 3^2 + 1 \cdot 3^1 + 2 \cdot 3^0 \\
2 \cdot 27 + 0 \cdot 9 + 1 \cdot 3 + 2 \cdot 1 \\
54 + 0 + 3 + 2 \\
59_{10}
\]
Convert $59_{10}$ from base-10 to base-3. Repeatedly divide by the greatest power of $b$ (the base) that is less than the number.

<table>
<thead>
<tr>
<th>Remainder</th>
<th>Try dividing</th>
<th>Digit</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>$3^4 = 81$</td>
<td>0</td>
<td>Too big</td>
</tr>
<tr>
<td>$59 - 0 \cdot 81 = 59$</td>
<td>$3^3 = 27$</td>
<td>2</td>
<td>O.K.</td>
</tr>
<tr>
<td>$59 - 2 \cdot 27 = 5$</td>
<td>$3^2 = 9$</td>
<td>0</td>
<td>Too big</td>
</tr>
<tr>
<td>$5 - 0 \cdot 9 = 5$</td>
<td>$3^1 = 3$</td>
<td>1</td>
<td>O.K.</td>
</tr>
<tr>
<td>$5 - 1 \cdot 3 = 2$</td>
<td>$3^0$</td>
<td>2</td>
<td>O.K.</td>
</tr>
</tbody>
</table>

$02012_3 = 2012_3$
Conversion works for any base

Review: For base-10, given an $n$-digit number in which $d_i$ is the $i$th digit, the number is

$$\sum_{i=1}^{n} 10^{i-1} \cdot d_i$$

For base-$b$, given an $n$-digit number in which $d_i$ is the $i$th digit, the number is

$$\sum_{i=1}^{n} b^{i-1} \cdot d_i$$
Useful bases

- **2**: Also called *binary*. Most fundamental base in digital logic. Know this like the back of your hand.
- **8**: Also called *octal*. Sometimes used by programmers. Prefer base 16.
- **10**: Also called *decimal or Arabic*.
- **16**: Also called *hexadecimal or simple hex*. One of the most compact and beautiful representations for digital computer programmers.
Binary

\[
\begin{array}{cccccccccc}
1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 & 512 & 1,024 (1K) \\
2^0 & 2^1 & 2^2 & 2^3 & 2^4 & 2^5 & 2^6 & 2^7 & 2^8 & 2^9 & 2^{10} \\
\end{array}
\]

\[k \neq K\]

\[
\begin{align*}
1k &= 10^3 &= 1,000 \\
1K &= 2^{10} &= 1,024
\end{align*}
\]
**Decimal**

- Most commonly used by human beings.
- Also called *Arabic*.
  - Actually developed in India and brought to Europe via Arabian empire.
- Largely replaced *Roman numerals*, which were more cumbersome when writing the large and complicated numbers used in astronomy and wide-spread trade.
Number systems

- Representation of positive numbers same in most systems
- A few special-purpose alternatives exist, e.g., Gray code
- Alternatives exist for signed numbers
Base-16: Hex

Often prefixed with 0x.
What is 0xFF?
Outline

1. Number Systems

2. Homework
Reading assignment

- Sections 5.1–5.6
Computer geek culture reference

- Spelling things in ASCII (hex or binary)
- This is one of the lower forms of geek culture, akin to bad puns
- However, at least one university has things written into its buildings with subtle brick patterns in ASCII binary

4a6934207375616e34206a6931207368653420
6a6934206865632068616f332077616e3221