## EECS 312 Homework 2 Solution

Q1:
Plasma Etching is more selective (directional etching, anisotropic) compared to wet etching (isotropic) however wet etching can be performed in batch mode (processing many wafers all at once).

## Q3:

By using the high $k$ dielectric and keeping all the other parameters constant we can increase transconductance compared to the case with a low $k$ dielectric material. $I^{\sim} C_{o x}$ and $C_{o x}=E_{o x} / t_{o x}$. By changing silicon dioxide ( SiO 2 ) with a high K dielectric material, let's say y we have $\mathrm{I} \sim \mathrm{C}_{\mathrm{y}}=\mathrm{E}_{\mathrm{y}} / \mathrm{T}_{\mathrm{y}}$ and $E_{y}>E_{o x}$. Therefore we have more current.

Q4:
A double-gate transistors on an SOI substrate in which the gate is wrapped around the channel.


## Q5:

In the question it says: $\mathrm{V}_{\mathrm{DD}}=\mathrm{V}_{\mathrm{t}}$; conclusion: this devices is either in cut off or linear region.
a) $\mathrm{C}_{\mathrm{gcs}}, \mathrm{C}_{\mathrm{gcd}}$ and $\mathrm{C}_{\mathrm{gcb}}$ contribute to $\mathrm{C}_{\mathrm{T}}$.
b) $\mathrm{V}_{\mathrm{gs}}<\mathrm{V}_{\mathrm{t}}$ : Cut-off: $\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{gcb}}=\mathrm{WLC}_{\mathrm{ox}}$
$\mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{t}}<\mathrm{V}_{\mathrm{ds}}$ : Saturation: $\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{gcs}}=2 / 3 \mathrm{WLC}_{\mathrm{ox}}$
$\mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{t}}>\mathrm{V}_{\mathrm{ds}}$ : Linear: $\mathrm{C}_{\mathrm{T}}=\mathrm{C}_{\mathrm{gcs}}+\mathrm{C}_{\mathrm{gcd}}=1 / 2 \mathrm{WLC}_{\mathrm{ox}}+1 / 2 \mathrm{WLC}_{\mathrm{ox}}=\mathrm{WLC}_{\mathrm{ox}}$
c) $\mathrm{t}=\mathrm{t} 1+\mathrm{t} 2=\mathrm{C} 1 \Delta \mathrm{~V} 1 / I_{\text {in }}+\mathrm{C} 2 \Delta \mathrm{~V} 2 / \mathrm{I}_{\text {in }}=\mathrm{WLC} \mathrm{ox} \cdot \mathrm{V}_{\mathrm{t}} / \mathrm{I}_{\text {in }}+2 / 3 \cdot \mathrm{WLC}_{\mathrm{ox}} \cdot\left(2 \mathrm{~V}_{\mathrm{t}}-\mathrm{V}_{\mathrm{t}}\right) / \mathrm{I}_{\text {in }}+\mathrm{WLC}_{\mathrm{ox}} \cdot\left(3 \mathrm{~V}_{\mathrm{t}}-2 \mathrm{~V}_{\mathrm{t}}\right) / I_{\text {in }}$
$=8 / 3 \cdot W L C_{o x} \cdot V_{t} / l_{\text {in }}$

Q6:

$V_{t A}=-0.4 V V_{D D}=2.5 \mathrm{~V}$
(a) $\mathrm{V}_{\text {in }}=0 \mathrm{~V}$; this implies $B$ is off

No current flows through $A$, but is on since $V_{G S}=0>-0.4 V$
The mode of operation of $A$ is linear as $V_{G s}-V_{t}>V_{D S}=0 V$ (Vout=2.5V as it pulled to VDD)
(b) $\mathrm{V}_{\text {in }}=2.5 \mathrm{~V}$ and $B$ is on
(i) First assume both $A$ and $B$ is in linear region: $I_{D}=k_{n}{ }^{\prime} W / L\left[\left(V_{G S}-V_{t}\right) V_{D S}-V_{D S}{ }^{2} / 2\right]$

Setting two current in two devices equal:
$k_{n}{ }^{\prime} W / L\left[\left(V_{G S_{-} B}-V_{t_{-} B}\right) V_{D S_{-} B}-V_{D S_{-} B}{ }^{2} / 2\right]=k_{n}{ }^{\prime} W / L\left[\left(V_{G S_{-} A}-V_{t_{-} A}\right) V_{D S_{-} A}-V_{D S_{-} A}{ }^{2} / 2\right]$
$(2.5-0.4) \mathrm{V}_{\text {out }}-\mathrm{V}_{\text {out }}{ }^{2} / 2=0.4\left(2.5-\mathrm{V}_{\text {out }}\right)-\left(2.5-\mathrm{V}_{\text {out }}\right)^{2} / 2$
$\rightarrow$ Doesn't solve
(ii) Assume $B$ is in saturation while $A$ is in linear:
$1 / 2 k_{n}{ }^{\prime} W / L\left(V_{G S_{-} B}-V_{t_{-} B}\right)^{2}=k_{n}{ }^{\prime} W / L\left[\left(V_{G S_{-A}}-V_{t_{-} A}\right) V_{D S_{-} A}-V_{\left.D S_{-}{ }^{2} / 2\right]}\right.$
$1 / 2(2.5-0.4)^{2}=0.4\left(2.5-V_{\text {out }}\right)-\left(2.5-V_{\text {out }}\right)^{2} / 2$
Vout $=2.1+2.062 i, 2.1+2.062 i \rightarrow$ Complex, Not good.
(iii) Assume $B$ is in linear while $A$ is in saturation:
$k_{n}{ }^{\prime} W / L\left[\left(V_{G S_{-} B}-V_{t_{-} B}\right) V_{D S_{-} B}-V_{D S_{-} B}{ }^{2} / 2\right]=1 / 2 k_{n}{ }^{\prime} W / L\left(V_{G S_{-} A}-V_{t_{-} A}\right)^{2}$
$(2.5-0.4) \mathrm{V}_{\text {out }}-\mathrm{V}_{\text {out }}{ }^{2} / 2=1 / 2(0.4)^{2}$
$\mathrm{V}_{\text {out }}=\underline{0.038 \mathrm{~V}}, 4.16 \mathrm{~V}\left(>\mathrm{V}_{\mathrm{DD}}\right)$
Check $B$ is in linear: 2.5-0.038 $>V_{t} \rightarrow$ o.k.
Check $A$ is in saturation: 0-(2.5-0.038) $<\mathrm{V}_{\mathrm{t}} \rightarrow$ o.k.
(c) $\quad \mathrm{P}_{\mathrm{avg}}=2.5 \mathrm{~V} \cdot \mathrm{I}_{\mathrm{D}} \cdot 0.7\left(\mathrm{~V}_{\mathrm{in}}=2.5 \mathrm{~V}\right)+2.5 \mathrm{~V} \cdot 0 \cdot 0.3\left(\mathrm{~V}_{\mathrm{in}}=0 \mathrm{~V}\right)$

$$
=2.5 \mathrm{~V} \cdot\left(1 / 2 \mathrm{k}_{\mathrm{n}}^{\prime} \mathrm{W} / \mathrm{L}(0.4)^{2}\right) * 0.7=0.14 \cdot \mathrm{k}_{\mathrm{n}}^{\prime} \cdot \mathrm{W} / \mathrm{L} \text { Watt }
$$

Q7:
(a) $\quad P_{\text {dynamic }}=C V^{2} f_{0->1}$

$$
P_{\text {clock }}=10000 \cdot 40 \mathrm{fF} \cdot 2.5 \mathrm{~V}^{2} \cdot 1 \mathrm{GHz}=2.5 \mathrm{~W}
$$

$$
P_{\text {comb }}=5000000 \cdot 6 \mathrm{fF} \cdot 2.5 \mathrm{~V}^{2} \cdot(0.1 \cdot 1 \mathrm{GHz} \cdot 0.5)=9.375 \mathrm{~W} \rightarrow \mathrm{f}_{0->1}=\alpha_{\mathrm{sw}} \cdot 1 \mathrm{GHz} \cdot 0.5
$$

$\mathrm{P}_{\text {dynamic }}=\mathrm{P}_{\text {clock }}+\mathrm{P}_{\text {comb }}=11.875 \mathrm{~W}$
Time $_{\text {depletion }}=\mathrm{E}_{\text {battery }} / \mathrm{P}_{\text {dynamic }}=39 \mathrm{~W} \cdot \mathrm{hr} / 11.875 \mathrm{~W}=3.28$ hours
(b) $\quad \mathrm{I}_{\text {off }}\left(\mathrm{l}_{\mathrm{DS}} @ \mathrm{~V}_{G S}=0\right)$ is given as $10^{-10} \mathrm{~A}$ for $\mathrm{V}_{\mathrm{t}}=0.5 \mathrm{~V}$

Determine $I_{\text {off }}$ when $\mathrm{V}_{\mathrm{t}}=0.25 \mathrm{~V}$


Note that $\mathrm{V}_{\mathrm{t}}$ shift corresponds with a shift of $I_{D S}$ vs. $V_{G S}$ graph
Slope of sub-threshold region is $90 \mathrm{mV} / \mathrm{dec}$ and shifting the sub-threshold line left by 250 mV will increase the off current by $250 \mathrm{mV} / 90 \mathrm{mV}=2.78 \mathrm{dec}$ Therefore $\mathrm{I}_{\text {off }} @ \mathrm{~V}_{\mathrm{t}}=0.25 \mathrm{~V}$

$$
=10^{-10} * 10^{2.78}=10^{-7.22}=6.026 \cdot 10^{-8} \mathrm{~A}
$$

$P_{\text {static }}=I_{\text {static }} \cdot V$
$=5010000 \cdot 6.026 \cdot 10^{-8} \mathrm{~A} \cdot 2.5 \mathrm{~V}=0.75 \mathrm{~W}$
$P_{\text {total }}=P_{\text {dynamic }}+P_{\text {static }}$
$=11.875 \mathrm{~W}+0.75 \mathrm{~W}=12.625 \mathrm{~W}$
Time $_{\text {depletion }}=\mathrm{E}_{\text {battery }} / \mathrm{P}_{\text {total }}=39 \mathrm{~W} \cdot \mathrm{hr} / 12.625 \mathrm{~W}=3.09$ hours
$\%$ reduction $=(3.28-3.09) / 3.28=5.79 \%$
(C) From Fig. 5-17 @ $V_{D D}=1.3$ normalized $t_{p}$ is $\sim 2$. That is, propagation delay has doubled so peak frequency has to be halved. The operating clock frequency $=0.5 \cdot 1 \mathrm{GHz}=500 \mathrm{mHz}$

$$
\begin{aligned}
& \mathrm{P}_{\text {dynamic }}=\mathrm{P}_{\text {clock }}+\mathrm{P}_{\text {comb }}=10000 \cdot 4 \mathrm{OfF} \cdot 1.3 \mathrm{~V}^{2} \cdot 500 \mathrm{mHz}+5000000 \cdot 6 \mathrm{fF} \cdot 1.3 \mathrm{~V}^{2} \cdot 0.10 \cdot 500 \mathrm{mHz} \cdot 0.5 \\
&=1.61 \mathrm{~W} \\
& \mathrm{P}_{\text {static }}=5010000 \cdot 6.026 \cdot 10^{-8} \mathrm{~A} \cdot 1.3 \mathrm{~V}=0.391 \mathrm{~W} \\
& \mathrm{P}_{\text {total }}=\mathrm{P}_{\text {dynamic }}+\mathrm{P}_{\text {static }}=2.00 \mathrm{~W} \\
& \text { Time }_{\text {depletion }}=\mathrm{E}_{\text {battery }} / \mathrm{P}_{\text {total }}=39 \mathrm{~W} \cdot \mathrm{hr} / 2 \mathrm{~W}=19.5 \text { hours } \\
& \mathrm{P}_{\text {dynamic }}=\mathrm{P}_{\text {clock }}+\mathrm{P}_{\text {comb }}=10000 \cdot 40 \mathrm{fF} \cdot 2.5 \mathrm{~V}^{2} \cdot 500 \mathrm{mHz}+5000000 \cdot 6 \mathrm{fF} \cdot 2.5 \mathrm{~V}^{2} \cdot 0.10 \cdot 500 \mathrm{mHz} \cdot 0.5 \\
&=5.94 \mathrm{~W} \\
& \text { (d) } \quad \\
& \mathrm{P}_{\text {static }}=5010000 \cdot 6.026 \cdot 10^{-8} \mathrm{~A} \cdot 2.5 \mathrm{~V}=0.75 \mathrm{~W} \\
& \mathrm{P}_{\text {total }}=\mathrm{P}_{\text {dynamic }}+\mathrm{P}_{\text {static }}=6.69 \mathrm{~W} \\
& \text { Time } \\
& \text { depletion }=\mathrm{E}_{\text {battery }} / \mathrm{P}_{\text {total }}=39 \mathrm{~W} \cdot \mathrm{hr} / 6.69 \mathrm{~W}=5.83 \text { hours }
\end{aligned}
$$

Q10:
$V_{\text {instantaneous }}=V_{\text {final }}+\left(V_{\text {initial }}-V_{\text {final }}\right) e^{-t / R C}$
(a) By replacing the PMOSFET with $3.8 \mathrm{k} \Omega$,
$V_{D D} / 2=0-\left(V_{D D}-0\right) e^{-t / 3.8 k \Omega \cdot 73.2 f F}$
$\mathrm{t}=\ln 2 \cdot 3.8 \mathrm{k} \Omega \cdot 73.2 \mathrm{fF}=0.693 \cdot 2.782 \cdot 10^{-10}=192.81 \mathrm{ps}$
(b) PMOSFET will be cut-off when $\mathrm{V}_{G S}>\mathrm{V}_{\mathrm{t}}$ and for a PMOSFET, the terminal with the higher voltage is source between source and drain terminals. In this problem, the terminal connected to $C_{L}$ will have the higher voltage, so it works as a source terminal.

The PMOSFET will continue to discharge the $C_{L}$, but if $C_{L}$ is discharged to $\left|V_{T P}\right|=0.5 \mathrm{~V}$, the PMOSFET will be cut-off since $\mathrm{V}_{\mathrm{GS}}=0-\mathrm{V}_{\mathrm{S}}=-\mathrm{V}_{\mathrm{CL}}>\mathrm{V}_{\mathrm{t}}=-0.5 \mathrm{~V}$
Therefore the final voltage of the capacitor is $\left|\mathrm{V}_{\text {TP }}\right|=0.5 \mathrm{~V}$
(c) Since we assumed resistor-switch model, the final voltage is still 0 . In this equation, we assume there is a sudden change in resistance to infinity when PMOS cuts-off.
$0.5=0-\left(V_{D D}-0\right) e^{-t / 3.8 \mathrm{k} \Omega \cdot 73.2 f \mathrm{~F}}$
$\mathrm{t}=\ln 5 \cdot 3.8 \mathrm{k} \Omega \cdot 73.2 \mathrm{fF}=0.693 \cdot 2.782 \cdot 10^{-10}=447.68 \mathrm{ps}$
(d) Resistor-switch model assumes a constant resistance ( $3.8 \mathrm{~K} \Omega$ ) during a transition before cut-off and a step change in resistance to infinity as the PMOSFET becomes cut-off. However, with a real PMOSFET, on-resistance increases gradually with decreasing |Vgs|. Since $R$ is a function of time that is monotonically increasing, RC time constant and delay will increase over a transition. Therefore, the delay calculated with resistor-switch model may be optimistic. i.e., with a real PMOSFET, it takes more time to get to the final voltage due to long tailed transition.

