EECS 312: Digital Integrated Circuits
Final Exam

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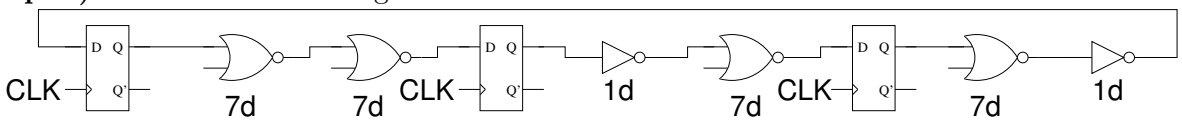
Show your work. Derivations are required for credit; end results are insufficient.
Closed book. You may use a calculator.

Honor Pledge: I have neither given nor received aid in this exam.

Signature:

1. **(10 pts.)** $k_n = \frac{W\mu_n\epsilon_{ox}}{Lt_{ox}}$ When the move from SiO_2 to high-κ gate dielectric occurred, what qualitative changes were made to the variables upon which k_n depends? You may assume that k_n remains fixed. Use only a few sentence fragments for your answer.

2. **(10 pts.)** Consider the following circuit:



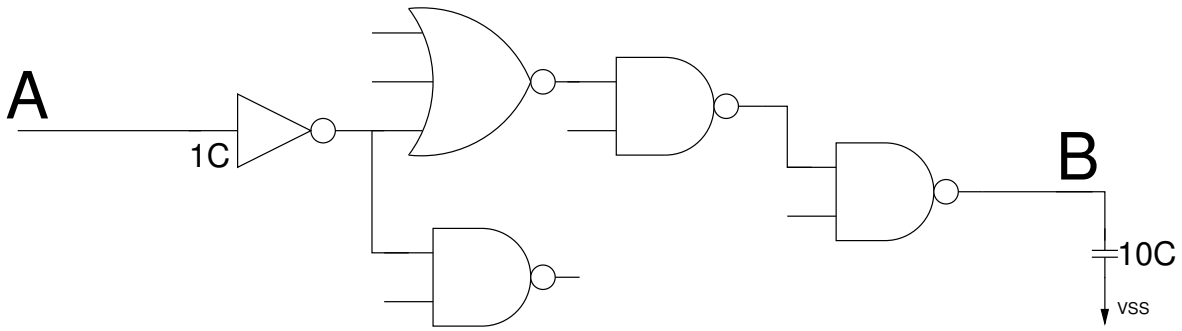
   The numbers near the logic gates indicate delays. Disconnected logic gate inputs may be assumed to be stable. Draw the gate- and latch-level diagram of a two-phase latch-based reimplementation of this circuit and indicate the minimal safe clock period. Do not change anything but the positions of the latches. You may neglect latch delays.

3. **(10 pts.)** Indicate two ways of programming floating-gate transistors and two ways of erasing floating-gate transistors. It is fine for the methods to require slightly different floating gate transistor designs. Use at most four short phrases.

4. **(10 pts.)** Draw the transistor-level diagram for a sense amplifier that reprograms the read value. Your circuit should have the following inputs: *read–program*, *equalize*, *bit*, and $\overline{bit}$. You may not assume access to complemented input literals.

5. **(10 pts.)** Draw a gate-level diagram for a three-bit carry-lookahead adder. Correctness counts the most, but efficiency also counts. You may use hierarchy. In other words, once you show how to construct something more complex than a standard logic gate, you can assign a symbol to it and reuse that symbol.

6. **(10 pts.)** Use the concept of logical effort to size the gates in the following circuit to minimize the delay for path $A \rightarrow B$.



You need not round your sizes.

7. **(10 pts.)** Draw a transistor-level diagram of a 3:8 decoder in which the transistors are sized to achieve the same output resistance to ground and V_{DD} as an inverter with gate capacitance $3C$.

8. **(10 pts.)** Draw the gate-level diagram of a rising edge triggered scan-flop, i.e., a flip-flop that can be used to support normal operation and scan-chain testing. You may use transistors in addition to logic gates. If sizing is essential for correct operation, label the appropriate gates and/or transistors as "narrow" and "wide".

9. **(10 pts.)** Determine the worst-case delay from precharge disable to output transition for a single-bit cell in a four-word MOS NAND ROM. λ =125 nm. All gates are 2λ long and 4λ wide. Sources and drains are 3λ long and 4λ wide. Assume (1) a default 0.25 µm process, (2) all inputs experiences a perfect step voltage change, and (3) an output capacitance of 10 fF.

	C_{OX} (fF/µm^2)	C_O (fF/µm)	C_j (fF/µm^2)	m_j	ϕ_b (V)	C_{jsw} (fF/µm)	m_{jsw}	ϕ_{bsw} (V)
NMOS	6	0.31	2	0.5	0.9	0.28	0.44	0.9
PMOS	6	0.27	1.9	0.48	0.9	0.22	0.32	0.9

MODELS FOR CMOS DEVICES

CMOS (0.25 μm) – Unified Model.

	V_{T0} (V)	γ ($V^{0.5}$)	V_{DSAT} (V)	k' (A/V^2)	λ (V^{-1})
NMOS	0.43	0.4	0.63	115×10^{-6}	0.06
PMOS	–0.4	–0.4	–1	-30×10^{-6}	–0.1

CMOS (0.25 μm) – Switch Model (R_{eq})

V_{DD} (V)	1	1.5	2	2.5
NMOS (kΩ)	35	19	15	13
PMOS (kΩ)	115	55	38	31

CMOS (0.25 μm) – BSIM Model

See Website: *http://bwrc.eecs.berkeley.edu/IcBook*

VALUES OF MATERIAL AND PHYSICAL CONSTANTS

Name	Symbol	Value	Units
Room temperature	T	300 (= 27°C)	K
Boltzman constant	k	1.38×10^{-23}	J/K
Electron charge	q	1.6×10^{-19}	C
Thermal voltage	$\phi_T = kT/q$	26	mV (at 300 K)
Intrinsic Carrier Concentration (Silicon)	n_i	1.5×10^{10}	cm^{-3} (at 300 K)
Permittivity of Si	ε_{si}	1.05×10^{-12}	F/cm
Permittivity of SiO_2	ε_{ox}	3.5×10^{-13}	F/cm
Resistivity of Al	ρ_{Al}	2.7×10^{-8}	Ω-m
Resistivity of Cu	ρ_{Cu}	1.7×10^{-8}	Ω-m
Magnetic permeability of vacuum (similar for SiO_2)	μ_0	12.6×10^{-7}	Wb/Am
Speed of light (in vacuum)	c_0	30	cm/nsec
Speed of light (in SiO_2)	c_{ox}	15	cm/nsec

FORMULAS AND EQUATIONS

Diode

$$I_D = I_S(e^{V_D/\phi_T} - 1) = Q_D/\tau_T$$

$$C_j = \frac{C_{j0}}{(1 - V_D/\phi_0)^m}$$

$$K_{eq} = \frac{-\phi_0^m}{(V_{high} - V_{low})(1-m)} \times [(\phi_0 - V_{high})^{1-m} - (\phi_0 - V_{low})^{1-m}]$$

MOS Transistor

$$V_T = V_{T0} + \gamma(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|})$$

$$I_D = \frac{k'_n}{2}\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) \text{ (sat)}$$

$$I_D = \upsilon_{sat} C_{ox} W\left(V_{GS} - V_T - \frac{V_{DSAT}}{2}\right)(1 + \lambda V_{DS}) \text{ (velocity sat)}$$

$$I_D = k'_n \frac{W}{L}\left((V_{GS} - V_T)V_{DS} - \frac{{V_{DS}}^2}{2}\right) \text{ (triode)}$$

$$I_D = I_S e^{\frac{V_{GS}}{nkT/q}}\left(1 - e^{-\frac{V_{DS}}{kT/q}}\right) \text{ (subthreshold)}$$

Deep Submicron MOS Unified Model

$$I_D = 0 \text{ for } V_{GT} \le 0$$

$$I_D = k'\frac{W}{L}\left(V_{GT}V_{min} - \frac{V_{min}^2}{2}\right)(1 + \lambda V_{DS}) \text{ for } V_{GT} \ge 0$$

with $V_{min} = \min(V_{GT}, V_{DS}, V_{DSAT})$

and $V_{GT} = V_{GS} - V_T$

MOS Switch Model

$$R_{eq} = \frac{1}{2}\left(\frac{V_{DD}}{I_{DSAT}(1 + \lambda V_{DD})} + \frac{V_{DD}/2}{I_{DSAT}(1 + \lambda V_{DD}/2)}\right) \approx \frac{3}{4}\frac{V_{DD}}{I_{DSAT}}\left(1 - \frac{5}{6}\lambda V_{DD}\right)$$

Inverter

$$V_{OH} = f(V_{OL})$$

$$V_{OL} = f(V_{OH})$$

$$V_M = f(V_M)$$

$$t_p = 0.69 R_{eq} C_L = \frac{C_L(V_{swing}/2)}{I_{avg}}$$

$$P_{dyn} = C_L V_{DD} V_{swing} f$$

$$P_{stat} = V_{DD} I_{DD}$$

Static CMOS Inverter

$$V_{OH} = V_{DD}$$

$$V_{OL} = GND$$

$$V_M \approx \frac{rV_{DD}}{1+r} \quad \text{with} \quad r = \frac{k_p V_{DSATp}}{k_n V_{DSATn}}$$

$$V_{IH} = V_M - \frac{V_M}{g} \qquad V_{IL} = V_M + \frac{V_{DD} - V_M}{g}$$

with $g \approx \frac{1+r}{(V_M - V_{Tn} - V_{DSATn}/2)(\lambda_n - \lambda_p)}$

$$t_p = \frac{t_{pHL} + t_{pLH}}{2} = 0.69 C_L\left(\frac{R_{eqn} + R_{eqp}}{2}\right)$$

$$P_{av} = C_L V_{DD}^2 f$$

Interconnect

Lumped *RC*: $t_p = 0.69\,RC$

Distributed *RC*: $t_p = 0.38\,RC$

RC-chain:

$$\tau_N = \sum_{i=1}^{N} R_i \sum_{j=i}^{N} C_j = \sum_{i=1}^{N} C_i \sum_{j=1}^{i} R_j$$

Transmission line reflection:

$$\rho = \frac{V_{refl}}{V_{inc}} = \frac{I_{refl}}{I_{inc}} = \frac{R - Z_0}{R + Z_o}$$

CMOS COMBINATIONAL LOGIC

Transistor Sizing using Logical Effort

$$F = \frac{C_L}{C_{g1}} = \prod_1^N \frac{f_i}{b_i} \qquad G = \prod_1^N g_i \qquad D = t_{p0}\sum_{j=1}^{N}\left(p_j + \frac{f_j g_j}{\gamma}\right)$$

$$B = \prod_1^N b_i \qquad H = FGB \qquad D_{min} = t_{p0}\left(\sum_{j=1}^{N} p_j + \frac{N(\sqrt[N]{H})}{\gamma}\right)$$