Introduction to Computer Engineering – EECS 203

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Outline

Review: Quine–McCluskey two-level logic minimization

- Compute prime implicants with a well-defined algorithm
  - Start from minterms
  - Merge adjacent implicants until further merging impossible
- Select minimal cover from prime implicants
  - Unate covering problem
- What is happening?
  - $ab + a\overline{b} = a$

Definition: Unate covering

Given a matrix for which all entries are 0 or 1, find the minimum cardinality subset of columns such that, for every row, at least one column in the subset contains a 1.

I'll give an example

Prime implicant selection

Prime implicants

Use these to...

On-set minterms	01X	0X0	X00	X11
000		1	1	
010	1	1		
011	1			1
111				1
100			1	

... cover these

Prime implicant selection

Prime implicants

Use these to...

On-set minterms

... cover these

	01X	0X0	X00	X11
~~000~~		1	1	
010	1	1		
011	1			1
111				1
~~100~~			1	

Prime implicant selection

Prime implicants

Use these to...

On-set minterms

... cover these

	01X	0X0	X00	X11
000		1	1	
010	1	1		
011	1			1
111				1
100			1	

Prime implicant selection

Prime implicants

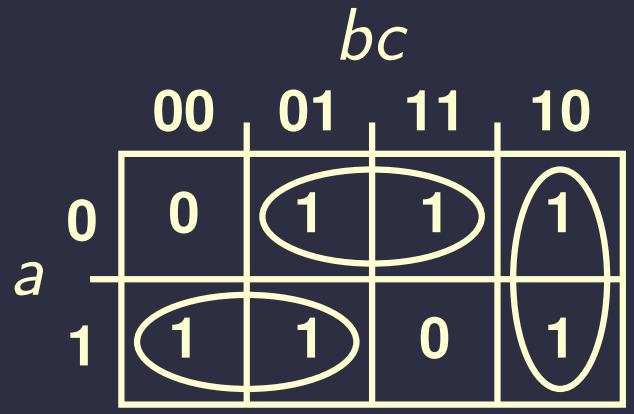
Use these to...

On-set minterms

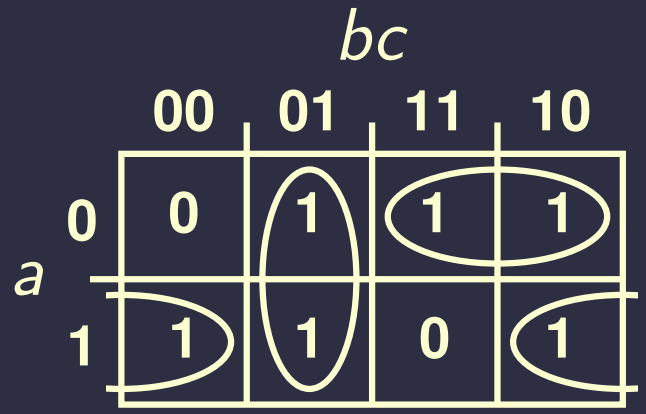
... cover these

	01X	0X0	X00	X11
~~000~~		1	1	
~~010~~	1	1		
~~011~~	1			1
~~111~~				1
~~100~~			1	

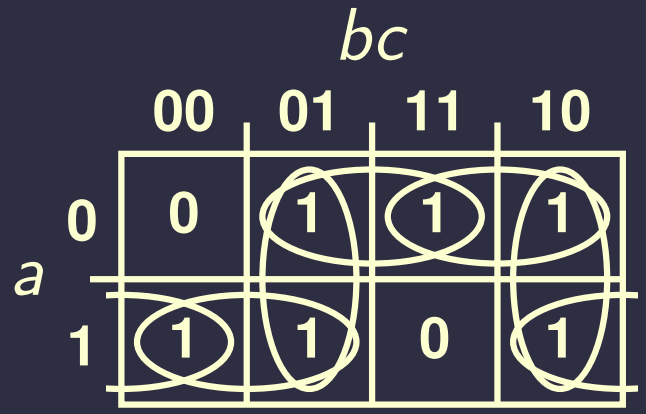
Cyclic core



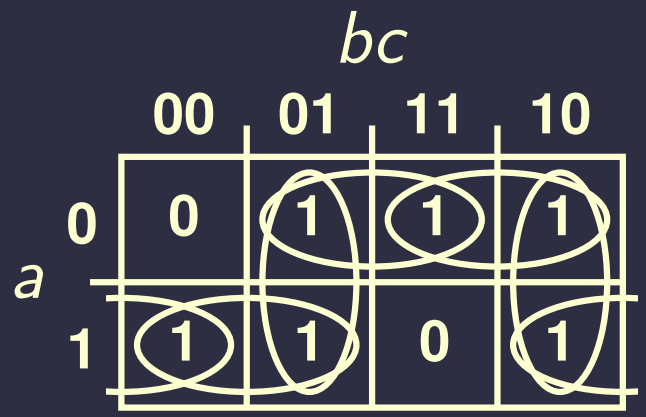
Cyclic core



Cyclic core

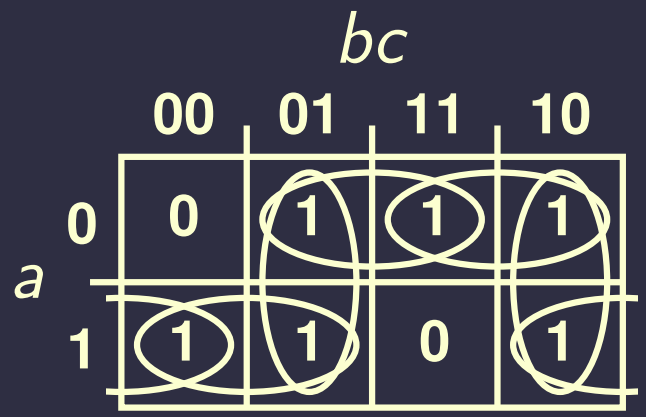


Cyclic core



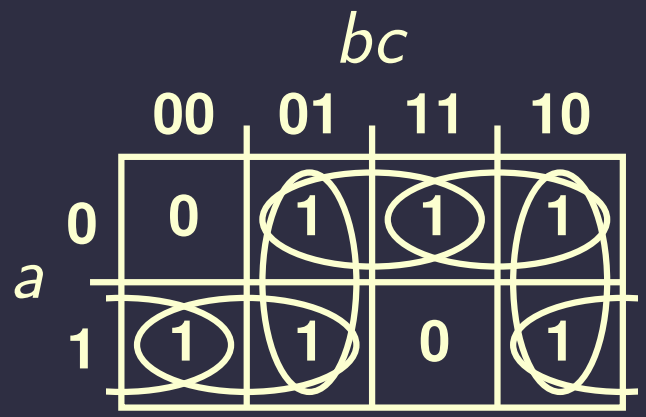
	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core



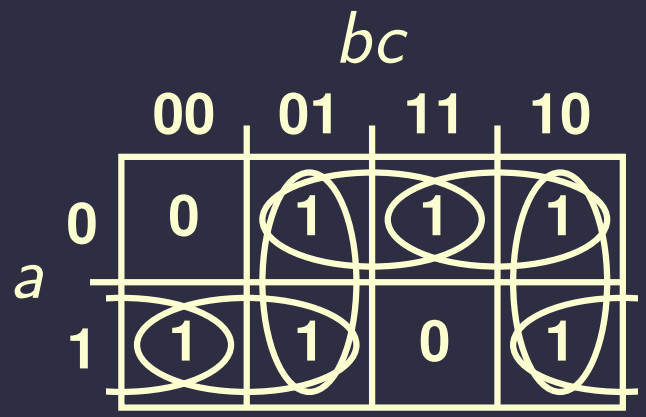
	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core



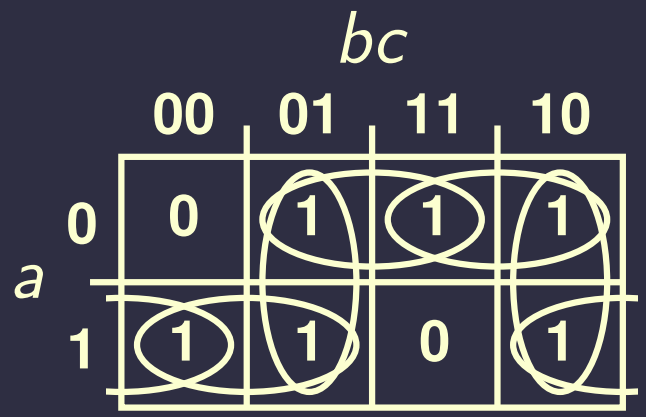
	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core



	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Cyclic core



	0X1	01X	X01	X10	10X	1X0
001	1		1			
011	1	1				
010		1		1		
100					1	1
101			1		1	
110				1		1

Implicant selection reduction

- Eliminate rows covered by essential columns
- Eliminate rows dominated by other rows
- Eliminate columns dominated by other columns

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows covered by essential columns

	A	B	C
H		1	
I	1		1
J	1	1	
K		1	1

Eliminate rows dominated by other rows

	A	B	C
H	1		
I	1	1	
J	1		1

Eliminate rows dominated by other rows

	A	B	C
H	1		
I	1	1	
J	1		1

Eliminate rows dominated by other rows

	A	B	C
H	1		
I	1	1	
J	1		1

Eliminate rows dominated by other rows

	A	B	C
H	1		
I	1	1	
J	1		1

Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

Eliminate columns dominated by other columns

	A	B	C
H	1		
I	1	1	
J	1		1
K		1	

Backtracking

- Will proceed to complete solution unless cyclic
- If cyclic, backtrack
  - Try all possible options to completion
- Advanced topic: Can use a number of tricks to simplify this

Use bound to constrain search space

- Eliminate rows covered by essential columns
- Eliminate rows dominated by other rows
- Eliminate columns dominated by other columns
- Speed improved, still $\in \mathcal{NP}$-complete
  - Too slow to solve for large problem instances

Loose end – Don't cares

- What should be done about Xs in QM?
- Should they be included in the initial minterms?
- Should they be required in the Unate Covering problem?

Another example

$$f(a, b, c) = \sum(1, 2, 6) + d(3)$$

Summary

- Review
- Prime implicant selection in Quine–McCluskey
- Encoders and decoders
- Review: Transmission gates
- Multiplexers and demultiplexers

Outline

Reading assignment

- M. Morris Mano and Charles R. Kime. *Logic and Computer Design Fundamentals*. Prentice-Hall, NJ, fourth edition, 2008
- Rest of Section 4.6

Computer geek culture reference

- Complexity classes
- Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman & Company, NY, 1979