

# Introduction to Computer Engineering – EECS 203

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**UNIVERSITY**

# Pace, lab expectations

- Anybody falling behind?
- If something isn't making sense, stop me and I'll elaborate using the chalkboard
  - I'm glad to do it!
- Lab expectations (lab two and above)
  - Complete schematics
  - Easy to debug, color-coded wiring
  - Terse but clear description

# Outline

1. The Quine–McCluskey two-level logic minimization method
2. Homework

# Review: Minimization techniques

## Advantages and disadvantages?

- Algebraic manipulation
- Karnaugh maps
- Quine–McCluskey
- Advanced topic: Kernel extraction
- Advanced topic: Heuristic minimization, e.g., Espresso

# Deriving POS

	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	0	1	1	1
10	1	1	0	1

Find SOP form for zeros:

$$\bar{f} = ab\bar{z} + \bar{c}d + \bar{a}\bar{b}d$$

# Deriving POS

Apply De Morgan's theorem

$$\bar{f} = ab\bar{d} + \bar{c}d + \bar{a}\bar{b}d \quad (1)$$

$$\bar{\bar{f}} = \overline{ab\bar{d} + \bar{c}d + \bar{a}\bar{b}d} \quad (2)$$

$$f = \overline{(ab\bar{d})} \cdot \overline{(\bar{c}d)} \cdot \overline{(\bar{a}\bar{b}d)} \quad (3)$$

$$f = (\bar{a} + \bar{b} + d)(c + \bar{d})(a + b + \bar{d}) \quad (4)$$

- Advanced topic: Read the POS expression directly from the Karnaugh map
  - More difficult

# Quine–McCluskey two-level logic minimization

- Compute prime implicants with a well-defined algorithm
  - Start from minterms
  - Merge adjacent implicants until further merging impossible
- Select minimal cover from prime implicants
  - Unate covering problem
- What is happening?
  - $ab + a\bar{b} = a$

# Computing prime implicants

---

$\Sigma = 0$     **0000**

---

$\Sigma = 1$     **0001**  
              **0010**  
              **1000**

---

$\Sigma = 2$     **1001**  
              **1010**

---

$\Sigma = 3$     **1101**  
              **1110**

---

$\Sigma = 4$     **1111**

---



# Computing prime implicants

---

$\Sigma = 0$       **0000**    **000X**

---

$\Sigma = 1$       **0001**  
                 **0010**  
                 **1000**

---

$\Sigma = 2$       **1001**  
                 **1010**

---

$\Sigma = 3$       **1101**  
                 **1110**

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$\Sigma = 4$       **1111**

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          **1010**

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          **1110**

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$\Sigma = 4$     **1111**

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# Computing prime implicants

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$\Sigma = 0$	<b>0000</b>	<b>000X</b>
		<b>00X0</b>

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$\Sigma = 1$	<b>0001</b>
	<b>0010</b>
	<b>1000</b>

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$\Sigma = 2$	<b>1001</b>
	<b>1010</b>

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$\Sigma = 3$	<b>1101</b>
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$\Sigma = 4$	<b>1111</b>
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# Computing prime implicants

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# Computing prime implicants

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$\Sigma = 0$	<b>0000</b>	<b>000X</b>
		<b>00X0</b>
		<b>X000</b>

---

$\Sigma = 1$	<b>0001</b>
	<b>0010</b>
	<b>1000</b>

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$\Sigma = 2$	<b>1001</b>
	<b>1010</b>

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# Computing prime implicants

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	<b>0000</b>	<b>000X</b>
$\Sigma = 0$		<b>00X0</b>
		<b>X000</b>

---

	<b>0001</b>
$\Sigma = 1$	<b>0010</b>
	<b>1000</b>

---

$\Sigma = 2$	<b>1001</b>
	<b>1010</b>

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$\Sigma = 3$	<b>1101</b>
	<b>1110</b>

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$\Sigma = 4$	<b>1111</b>
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---

# Computing prime implicants

---

	<b>0000</b>	<b>000X</b>
$\Sigma = 0$		<b>00X0</b>
		<b>X000</b>

---

	<b>0001</b>	<b>X001</b>
$\Sigma = 1$	<b>0010</b>	
	<b>1000</b>	

---

$\Sigma = 2$	<b>1001</b>
	<b>1010</b>

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# Computing prime implicants

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$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	
$\Sigma = 3$	<b>1101</b> <b>1110</b>	
$\Sigma = 4$	<b>1111</b>	



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$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	
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# Computing prime implicants

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$\Sigma = 0$	<b>0000</b>	<b>000X</b>
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		<b>X000</b>

---

$\Sigma = 1$	<b>0001</b>	<b>X001</b>
	<b>0010</b>	<b>X010</b>
	<b>1000</b>	

---

$\Sigma = 2$	<b>1001</b>
	<b>1010</b>

---

$\Sigma = 3$	<b>1101</b>
	<b>1110</b>

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$\Sigma = 4$	<b>1111</b>
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# Computing prime implicants

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$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	
$\Sigma = 3$	<b>1101</b> <b>1110</b>	
$\Sigma = 4$	<b>1111</b>	

# Computing prime implicants

---

	<b>0000</b>	<b>000X</b>
$\Sigma = 0$		<b>00X0</b>
		<b>X000</b>

---

	<b>0001</b>	<b>X001</b>
$\Sigma = 1$	<b>0010</b>	<b>X010</b>
	<b>1000</b>	<b>100X</b>

---

$\Sigma = 2$	<b>1001</b>
	<b>1010</b>

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$\Sigma = 3$	<b>1101</b>
	<b>1110</b>

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$\Sigma = 4$	<b>1111</b>
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# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b> <b>00X0</b> <b>X000</b>
$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b> <b>10X0</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	
$\Sigma = 3$	<b>1101</b> <b>1110</b>	
$\Sigma = 4$	<b>1111</b>	

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b> <b>00X0</b> <b>X000</b>
$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b> <b>10X0</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	
$\Sigma = 3$	<b>1101</b> <b>1110</b>	
$\Sigma = 4$	<b>1111</b>	

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b> <b>00X0</b> <b>X000</b>
$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b> <b>10X0</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	<b>1X01</b>
$\Sigma = 3$	<b>1101</b> <b>1110</b>	
$\Sigma = 4$	<b>1111</b>	

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b> <b>00X0</b> <b>X000</b>
$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b> <b>10X0</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	<b>1X01</b>
$\Sigma = 3$	<b>1101</b> <b>1110</b>	
$\Sigma = 4$	<b>1111</b>	



# Computing prime implicants

$\Sigma = 0$	0000	000X
		00X0
		X000
$\Sigma = 1$	0001	X001
	0010	X010
	1000	100X
		10X0
$\Sigma = 2$	1001	1X01
	1010	1X10
$\Sigma = 3$	1101	
	1110	
$\Sigma = 4$	1111	

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b> <b>00X0</b> <b>X000</b>
$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b> <b>10X0</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	<b>1X01</b> <b>1X10</b>
$\Sigma = 3$	<b>1101</b> <b>1110</b>	
$\Sigma = 4$	<b>1111</b>	

# Computing prime implicants

$\Sigma = 0$	0000	000X 00X0 X000
$\Sigma = 1$	0001 0010 1000	X001 X010 100X 10X0
$\Sigma = 2$	1001 1010	1X01 1X10
$\Sigma = 3$	1101 1110	111X
$\Sigma = 4$	1111	

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b> <b>00X0</b> <b>X000</b>
$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b> <b>10X0</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	<b>1X01</b> <b>1X10</b>
$\Sigma = 3$	<b>1101</b> <b>1110</b>	<b>111X</b>
$\Sigma = 4$	<b>1111</b>	

# Computing prime implicants

$\Sigma = 0$	0000	000X 00X0 X000
$\Sigma = 1$	0001 0010 1000	X001 X010 100X 10X0
$\Sigma = 2$	1001 1010	1X01 1X10
$\Sigma = 3$	1101 1110	111X 11X1
$\Sigma = 4$	1111	

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b> <b>00X0</b> <b>X000</b>
$\Sigma = 1$	<b>0001</b> <b>0010</b> <b>1000</b>	<b>X001</b> <b>X010</b> <b>100X</b> <b>10X0</b>
$\Sigma = 2$	<b>1001</b> <b>1010</b>	<b>1X01</b> <b>1X10</b>
$\Sigma = 3$	<b>1101</b> <b>1110</b>	<b>111X</b> <b>11X1</b>
$\Sigma = 4$	<b>1111</b>	

# Computing prime implicants

$\Sigma = 0$	0000	000X	X00X
		00X0	
		X000	
$\Sigma = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\Sigma = 2$	1001	1X01	
	1010	1X10	
$\Sigma = 3$	1101	111X	
	1110	11X1	
$\Sigma = 4$	1111		

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b>	<b>X00X</b>
		<b>00X0</b>	
		<b>X000</b>	
$\Sigma = 1$	<b>0001</b>	<b>X001</b>	
	<b>0010</b>	<b>X010</b>	
	<b>1000</b>	<b>100X</b>	
		<b>10X0</b>	
$\Sigma = 2$	<b>1001</b>	<b>1X01</b>	
	<b>1010</b>	<b>1X10</b>	
$\Sigma = 3$	<b>1101</b>	<b>111X</b>	
	<b>1110</b>	<b>11X1</b>	
$\Sigma = 4$	<b>1111</b>		



# Computing prime implicants

$\Sigma = 0$	0000	000X	X00X
		00X0	X0X0
		X000	
$\Sigma = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\Sigma = 2$	1001	1X01	
	1010	1X10	
$\Sigma = 3$	1101	111X	
	1110	11X1	
$\Sigma = 4$	1111		

# Computing prime implicants

$\Sigma = 0$	0000	000X	X00X
		00X0	X0X0
		X000	
$\Sigma = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\Sigma = 2$	1001	1X01	
	1010	1X10	
$\Sigma = 3$	1101	111X	
	1110	11X1	
$\Sigma = 4$	1111		

# Computing prime implicants

$\Sigma = 0$	0000	000X	X00X
		00X0	X0X0
		X000	
$\Sigma = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\Sigma = 2$	1001	1X01	
	1010	1X10	
$\Sigma = 3$	1101	111X	
	1110	11X1	
$\Sigma = 4$	1111		

# Computing prime implicants

$\Sigma = 0$	0000	000X	X00X
		00X0	X0X0
		X000	
$\Sigma = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\Sigma = 2$	1001	1X01	
	1010	1X10	
$\Sigma = 3$	1101	111X	
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# Computing prime implicants

$\Sigma = 0$	0000	000X	X00X
		00X0	X0X0
		X000	
$\Sigma = 1$	0001	X001	
	0010	X010	
	1000	100X	
		10X0	
$\Sigma = 2$	1001	1X01	
	1010	1X10	
$\Sigma = 3$	1101	111X	
	1110	11X1	
$\Sigma = 4$	1111		

# Computing prime implicants

$\Sigma = 0$	<b>0000</b>	<b>000X</b>	<b>X00X</b>
		<b>00X0</b>	<b>X0X0</b>
		<b>X000</b>	
$\Sigma = 1$	<b>0001</b>	<b>X001</b>	
	<b>0010</b>	<b>X010</b>	
	<b>1000</b>	<b>100X</b>	
		<b>10X0</b>	
$\Sigma = 2$	<b>1001</b>	<b>1X01</b>	
	<b>1010</b>	<b>1X10</b>	
$\Sigma = 3$	<b>1101</b>	<b>111X</b>	
	<b>1110</b>	<b>11X1</b>	
$\Sigma = 4$	<b>1111</b>		

# Summary

- Review: Minimization overview
- Review: Karnaugh map SOP minimization
- POS using SOP K-map trick
- Quine-McCluskey optimal two-level minimization method

## Next lecture – More advanced building blocks

- Encoders and decoders
- MUXs
- Advanced TG techniques



# Outline

1. The Quine–McCluskey two-level logic minimization method
2. Homework

## Reading assignment

- M. Morris Mano and Charles R. Kime. *Logic and Computer Design Fundamentals*. Prentice-Hall, NJ, third edition, 2004
- Sections 2.7–2.10
- Sections 4.1–4.5
- Section 4.6 (decoders and multiplexers only)

## Additional references

- If QM doesn't click, please also see the following references
- Randy H. Katz. *Contemporary Logic Design*. The Benjamin/Cummings Publishing Company, Inc., 1994: pp. 85–88
- John P. Hayes. *Introduction to Digital Logic Design*. Addison-Wesley, MA, 1993 pp. 320, 321
- You can get these from me or the library

# Computer geek culture reference

<http://www.deepchip.com/>